Rectangles with Different Shapes

What Happens

Students construct different-shaped rectangles with the same area, cutting apart the 10-by-12 grids and reassembling the pieces into new rectangles. They also work on the computer game, Tumbling Tetrominoes, with different-shaped rectangles. They discuss the "fairness" of the different shapes, touching on issues about conservation of area and strategies for covering. Their work focuses on:

- comparing areas of rectangles with different dimensions
- exploring how tetromino shapes fit in rectangles with different dimensions

Materials

- 10-by-12 Rectangle (6 per pair)
- Transparent or masking tape

For the computer activity:

- Students' Game Records (from earlier sessions)

Activity

Changing the Shape of the Rectangle

We've been trying to cover our 10-by-12 rectangle with all the different tetrominoes. You found out that some of the tetrominoes will cover it perfectly, but some don't. What if we changed the shape of the rectangle? Do you think the same shapes would work, and the same shapes not work?

We're going to try that, but first we're going to make some rectangles of different shapes to work with. We're looking for rectangles that have the same total area as the 10-by-12 rectangle—120 squares—but have a different shape.

Have students work in pairs to cut apart the 10-by-12 Rectangle into parts (smaller rectangles), then tape the parts back together to make a new rectangle with a different shape, using all the pieces. Encourage students to make several different rectangles containing 120 squares.

After they make a new rectangle, students must find a way to check that it has 120 squares. If you observe them counting squares one by one, help them think about ways to count more efficiently—that is, skip counting by rows (possibly with the help of a calculator). Display the completed rectangles, labeled with their dimensions.

See the **Dialogue Box,** Changing the Rectangle's Shape (p. 26), for a sample student discussion during this activity.

When several new rectangles have been displayed, ask students to reflect on their work. This is also a good time to introduce the term *dimensions* into the conversation.

The original rectangle has 10 squares in each row and 12 rows. So, its *dimensions* are 10 by 12. What are the dimensions for the new rectangles we made? Do they have the same number of squares? How can you tell how many squares are in your rectangle without counting by ones?

Note: Be sure students save their new rectangles for Session 5, when they will try covering them with tetrominoes.

Activity

Which Shapes Are Easiest to Cover?

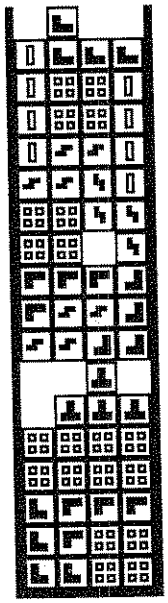
One of the options for Tumbling Tetrominoes, either on computer or in the paper version, is to choose a different-shaped rectangle to cover. The computer game offers several options beyond the original 12-by-10, including 10-by-12, 8-by-15, 15-by-8, 6-by-20, 20-by-6, 5-by-24, and 24-by-5.

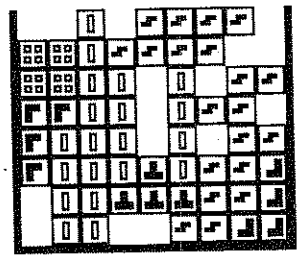
Note: All eight options will show on 13-inch screens. On 9-inch screens, the tall 6-by-20 and 5-by-24 rectangles will not fit, so these choices will not appear on the tool bar.

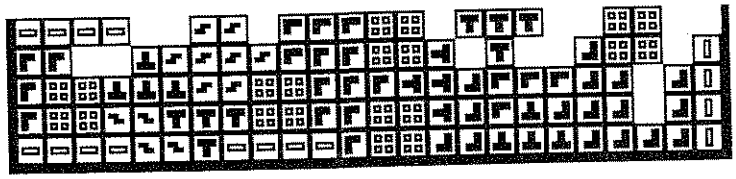
For the paper version, students can use as gameboards the new rectangles they taped together at the beginning of the session.

Before they begin, ask students to think about this:

Would one shape be easier to cover completely? Which one? Let's try it and find out.







Have small groups or individuals play the game. Some students may want to save their games on the computer to show what happened when they tried new shapes. You may want to observe while they work and choose a few games to save, especially shapes that were particularly hard to fill, as examples for the class to discuss.

After everyone has tried covering new rectangle shapes, discuss the following questions with the whole class:

Are the different shapes for the rectangles all "fair"? Was it easier to completely cover some shapes and harder to cover others?

Session 4 Follow-Up

Homework

Making More Rectangles Students can take home 10-by-12 Rectangles to try making additional rectangles of different shapes but with a total area of 120 square units.

Extension

Possible Scores Students can explore the question of which scores are possible in Tumbling Tetrominoes.

Some of you got perfect scores of 120. Many got 116 or 112. Could you get a score of 119? Why or why not? What scores are possible and which are impossible?

DIALOGUE BOX

Changing the Rectangle's Shape

As students started the activity Changing the Shape of the Rectangle (p. 23), many at first were not sure how to make a different-shaped rectangle from the basic 10-by-12 grid. The teacher encouraged them to persist, and eventually a few solutions emerged.

The first few ideas involved cutting the 10-by-12 rectangle in half, or into strips, and taping those together. After that, most students went on to construct several different rectangles. They then discussed how to check that the total number of squares remained 120.

Do the rectangles you made have the same number of little squares—the same area—as the 10-by-12 rectangle you started with?

Ly Dinh: Mine does. Because I just cut it in half and put it back together in a different way, so it's got to have the same number.

What are the new dimensions?

Ly Dinh: It's 1, 2, 3, 4, 5, by . . . oh! It must be 5 by 24, because there's two 12's on top of each other.

Mark: That's too skinny to be 120 in all.

How else could we figure it out?

Jennifer: We could count by 5's for each row.

Liliana: We could multiply 5 times 24.

Use your way to check all the rectangles we have made. Will they all have 120 small squares?

Liliana: I'm going to get a calculator and just multiply the two dimensions on every single rectangle.